Shape Reconstruction of a Buried Perfectly Conducting Cylinder by Inverse Schemes

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Abstract— Dynamic differential evolution (DDE) for shape reconstruction of perfect conducting cylinder (PEC) buried in a half-space is presented. Assume that a conducting cylinder of unknown shape is buried in one half-space and scatters the field incident from another half-space where the scattered filed is measured. Based on the boundary condition and the measured scattered field, a set of nonlinear integral equations is derived and the imaging problem is reformulated into an optimization problem. The inverse problem is resolved by an optimization approach, and the global searching scheme DDE is then employed to search the parameter space. Numerical results demonstrate that even when the initial guess is far away from the exact one, good reconstruction can be obtained by using DDE.

1. INTRODUCTION

The object of inverse electromagnetic scattering is to reconstruct the profile of unknown objects using measurement data. The inverse scattering problem has many applications in diverse fields such as nondestructive testing, medical imaging, geophysics, remote sensing, and ground penetrating radar [1–4].

As known, the inverse scattering problem is generally nonlinear and ill-posedness [5]. One way for solving the nonlinear inverse problem is solving the forward scattering problem iteratively to minimize an error function known as the cost function. This function represents the error between the measured scattered fields and the simulated fields during the updates of the evolving objects in each inversion. Recently, several papers for inverse scattering problems have been published on the subject of 2-D object about deal with shape reconstruction problems by using genetic algorithms (GAs) [6–10], differential evolution (DE) [11–13], particle swarm optimization (PSO) [14–18] and Neural network [19].

In the 2010, the dynamic differential evolution (DDE) was first proposed to deal with the shape reconstruction of homogenous dielectric objects [11]. DDE algorithm is potentially able to obtain the global optimum of a functional whatever the initial guesses are. It is also found that DDE algorithms have good convergences compared with PSO methods in the inverse scattering problems [20]. To the best of our knowledge, there is still no investigation on using the DDE to reconstruct the electromagnetic imaging of buried perfect conducting cylinders.

In this paper, the goal of the current work is to reconstruct 2-D buried PEC target with arbitrary cross sections, and dynamic differential evolution (DDE) is employed to recover the shape of a buried perfectly conducting cylinder. In Section 2, a theoretical formulation for the inverse scattering is presented. The general principles of DDE, we applied them to the inverse problem are described in Section 3. Numerical results for reconstructing objects of different shapes with noise are given in Section 4. Finally, some conclusions are drawn in Section 5.

2. THEORETICAL FORMULATION

Let us consider a perfectly conducting cylinder which is buried in a lossy homogeneous half-space, as shown in Figure 1. Media in regions 1 and 2 are characterized by permittivity and conductivity $(\varepsilon_1, \sigma_1)$ and $(\varepsilon_2, \sigma_2)$, respectively and the permeability in both regions are μ_0 , i.e., non magnetic

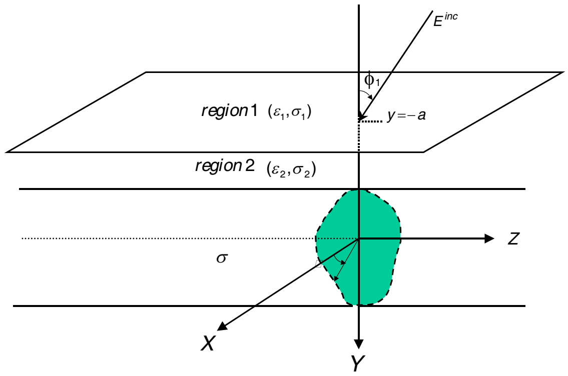

Figure 1: Geometry of the problem in (x, y) plane.

media are concerned here. The cross section of the cylinder are described in polar coordinates in xy plane by the equation $\rho = F(\theta)$. The cylinder is illuminated by a plane wave with time dependence $e^{j\omega t}$.

For simplicity, the electric field vector is assumed to be parallel to the z-axis (i.e., transverse magnetic or TM polarization). Let E^{inc} denote the incident field from region 1 with incident angle ϕ_1. Owing to the interface between region 1 and region 2, the incident plane wave generates two waves which would exist in the absence of the conducting object: a reflected wave (for $y \le -a$) and a transmitted wave (for $y > -a$). Thus unperturbed field is given by

$$\vec{E}_i(\vec{r}) = E_i(x, y)\hat{z} \tag{1}$$

For a TM incident wave, the scattered field can be expressed as

$$E_s(x, y) = -\int_0^{2\pi} G(x, y; F(\theta'), \theta')J(\theta')d\theta' \tag{2}$$

with

$$\begin{aligned} J(\theta) &= -j\omega\mu_0\sqrt{F^2(\theta) + F'^2(\theta)}J_s(\theta) \\ G(x, y; x', y') &= \begin{cases} G_1(x, y; x', y'), & y \le -a \\ G_2(x, y; x', y') = G_f(x, y; x', y') + G_s(x, y; x', y'), & y > -a \end{cases} \end{aligned} \tag{3}$$

where

$$G_1(x, y; x', y') = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{j}{\gamma_1 + \gamma_2} e^{j\gamma_1(y+a)}e^{-j\gamma_2(y'+a)}e^{-j\alpha(x-x')}d\alpha \tag{3a}$$

$$G_f(x, y; x', y') = \frac{j}{4}H_0^{(2)}\left[k_2\sqrt{(x - x')^2 + (y - y')^2}\right] \tag{3b}$$

$$G_s(x, y; x', y') = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{j}{2\gamma_2}\left(\frac{\gamma_2 - \gamma_1}{\gamma_2 + \gamma_1}\right) e^{-j\gamma_2(y+2a+y)}e^{-j\alpha(x-x')}d\alpha \tag{3c}$$

$$\gamma_i^2 = k_i^2 - \alpha^2, \quad i = 1, 2, \quad \mathrm{Im}(\gamma_i) \le 0, \quad y' > a$$

Here $J_s(\theta)$ is the induced surface current density which is proportional to the normal derivative of electric field on the conductor surface. $G(x, y; x', y')$ is the Green's function which can be obtained by Fourier transform. In (3b), $H_0^{(2)}$ is the Hankel function of the second kind of order zero.

Let us consider the following inverse problem: giving the scattered electric field E_s measured outside the scatterer determine the shape $F(\theta)$ of the object. Assume the approximate center of the scatterer, which in fact can be any point inside the scatterer, is known. Then the shape function $F(\theta)$ can be expanded as:

$$F(\theta) \cong \sum_{n=0}^{\frac{N}{2}} B_n\cos(n\theta) + \sum_{n=1}^{\frac{N}{2}} C_n\sin(n\theta) \tag{4}$$

where B_n and C_n are real coefficients to be determined, and $N + 1$ is the number of unknowns.

For the inverse scattering problem, the shape of the PEC is reconstructed by the given scattered electric field measured at the receivers. This problem is resolved by an optimization approach, for which the global searching scheme DDE is employed to minimize the following objective function (OF):

$$OF = \left\{ \frac{1}{M_t} \sum_{m=1}^{M_t} \left| E_s^{\exp}(\bar{r}_m) - E_s^{cal}(\bar{r}_m) \right|^2 / \left| E_s^{\exp}(\bar{r}_m) \right|^2 \right\}^{1/2} \tag{5}$$

where M is the total number of measured points. $E_s^{\exp}(\vec{r}_m)$ and $E_s^{cal}(\vec{r}_m)$ are the measured scattered field and the calculated scattered field respectively. Therefore, the maximization of OF can be interpreted as the minimization of the least-squares error between the measured and the calculated fields. It should be noted that the shape function used to describe the shape of the cylinder will be determined by the DDE scheme.

3. DYNAMIC DIFFERENTIAL EVOLUTION (DDE)

DDE algorithm starts with an initial population of potential solutions that is composed by a group of randomly generated individuals which represents shape function of the cylinders. Each individual in DDE algorithm is aD-dimensional vector consisting of D optimization parameters. The initial population may be expressed by $\{x_i : \quad i = 1,\ 2, \cdots, Np\}$, where Np is the population size. After initialization, DDE algorithm performs the genetic evolution until the termination criterion is met. DDE algorithm, like other EAs, also relies on the genetic operations (mutation, crossover and selection) to evolve generation by generation.

The key distinction between a DDE algorithm and a typical DE [21] is on the population updating mechanism. In a typical DE, all the update actions of the population are performed at the end of the generation, of which the implementation is referred as static updating mechanism. Alternatively, the updating mechanism of a DDE algorithm is carried out in a dynamic way: each parent individual will be replaced by his offspring if the offspring has a better objective function value than its parent individual does. Thus, DDE algorithm can respond the progress of population status immediately and to yield faster convergence speed than the typical DE. Based on the convergent characteristic of DDE algorithm, we are able to reduce the numbers of objective function evaluation and reconstruct the microwave image efficiently.

4. NUMERICAL RESULTS

Let us consider a perfectly conducting cylinder which is buried in a lossless half-space ($\sigma_1 = \sigma_2 = 0$). the permittivity in region 1 and region 2 is characterized by $\varepsilon_1 = \varepsilon_0$ and $\varepsilon_2 = 2.7\varepsilon_0$, respectively. A TM polarization plane wave of unit amplitude is incident from region 1 upon the object as shown in Fig. 1. The frequency of the incident wave is chosen to be 3 GHz, i.e., the wavelength λ_0 is 0.1 m. The object is buried at a depth $a = \lambda_0$ and the scattered field is measured on a probing line along the interface between region 1 and region 2. Our purpose is to reconstruct the shape of the object by using the scattered field at different incident angles. To reconstruct the shape of the object, the object is illuminated by incident waves from three different directions and 8 measurement points at equal spacing are used along the interface $y = -a$ for each incident angle. There are 24 measurement points in each simulation. To save computing time, the number of unknowns is set to be 7. The parameters and the corresponding searching ranges are listed follows: The operational coefficients are set as below: The crossover rate CR is set to be 0.8. Both parameters F and λ are set to be 0.8. The population size Np is set to be 110. The search range for the unknown coefficient of the shape function is chosen to be from 0 to 0.1. The extreme value of the coefficient of the shape function can be determined by the prior knowledge of the objects. Here DR, which is called shape function discrepancies, is defined as

$$DR = \left\{ \frac{1}{N'} \sum_{i=1}^{N'} [F^{cal}(\theta_i) - F(\theta_i)]^2 / F^2(\theta_i) \right\}^{1/2} \tag{6}$$

In the example, the shape function is chosen to be $F(\theta) = 0.03 + 0.01\cos(2\theta)$ m. The reconstructed images for different generations and the relative error of the example are shown in Figure 2 and Figure 3, respectively. Figure 3 shows that the relative errors of the shape decrease quickly and good convergences are achieved within 50 generation. The DR value is about 0.5% in the final generation.

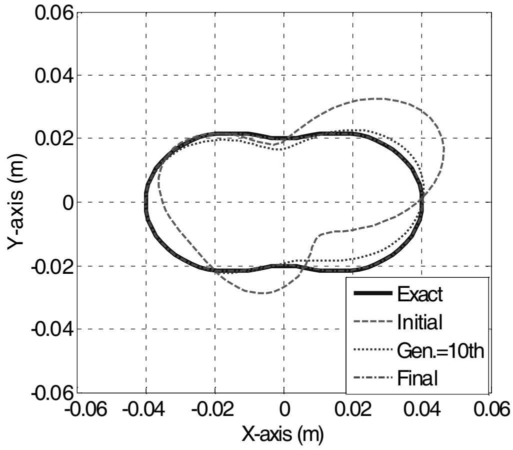

Figure 2: The reconstructed shape of the cylinder at different generations for the example.

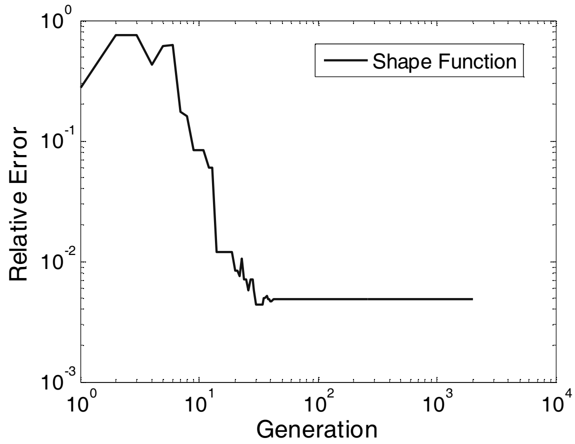

Figure 3: Shape function error versus generation for the example.

5. CONCLUSION

We have presented a study of applying the DDE to reconstruct the shape of a buried conducting cylinder. These approaches are applied to two-dimensional configurations. After an integral formulation, a discretization using the method of moment (MoM) is applied. Considering that the microwave imaging is recast as a nonlinear optimization problem, objective function is defined by the norm of a difference between the measured scattered electric field and the calculated scattered field for an estimated the shape of metallic cylinder. Thus, the shape of metallic cylinder can be obtained by minimizing the objective function.

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REFERENCES

1. Mudanyalı, O., S. Yıldız, O. Semerci, A. Yapar, and I. Akduman, "A microwave tomographic approach for nondestructive testing of dielectric coated metallic surfaces," *IEEE Geoscience and Remote Sensing Letters*, Vol. 5, No. 2, 180–184, Apr. 2008.
2. Benedetti, M., D. Lesselier, M. Lambert, and A. Massa, "Multiple-shape reconstruction by means of multiregion level sets," *IEEE Transactions Geoscience and Remote Sensing*, Vol. 48, No. 5, 2330–2342, May 2010.
3. Sun, C. H., C. C. Chiu, and C. J. Lin, "Image reconstruction of inhomogeneous biaxial dielectric cylinders buried in a slab medium," *International Journal of Applied Electromagnetics and Mechanics*, Vol. 34, No. 1–2, 33–48, Nov. 2010.
4. Sun, C. H., C. L. Li, C. C. Chiu, and C. H. Huang, "Time domain image reconstruction for a buried 2D homogeneous dielectric cylinder using NU-SSGA," *Research in Nondestructive Evaluation*, Vol. 22, No. 1, 1–15, Jan. 2011.
5. Sabatier, P. C., "Theoretical considerations for inverse scattering," *Radio Science*, Vol. 18, 629–631, Jan. 1983.
6. Sun, C. H., C. L. Liu, K. C. Chen, C. C. Chiu, C. L. Li, and C. C. Tasi, "Electromagnetic transverse electric wave inverse scattering of a partially immersed conductor by steady-state genetic algorithm," *Electromagnetics*, Vol. 28, No. 6, 389–400, Aug. 2008.
7. Pastorino, M., A. Massa and S. Caorsi, "A microwave inverse scattering technique for image reconstruction based on a genetic algorithm," *IEEE Transactions on Instrumentation and Measurement*, Vol. 49, No. 3, 573–578, 2000.
8. Caorsi, S., A. Massa, and M. Pastorino, "A computational technique based on a real-coded genetic algorithm for microwave imaging purposes," *IEEE Transactions on Geoscience and Remote Sensing, Imaging and Target Identification*, Vol. 38, No. 4, 1697–1708, 2000.
9. Caorsi, S., A. Massa, and M. Pastorino, "A crack identification microwave procedure based on a genetic algorithm for nondestructive testing," *IEEE Transactions on Antennas and Propagation*, Vol. 49, No. 12, 1812–1820, 2001.

10. Chien, W., C. H. Sun, and C. C. Chiu, "Image Reconstruction for a partially immersed imperfectly conducting cylinder by genetic algorithm," *International Journal of Imaging Systems and Technology*, Vol. 19, 299–305, Dec. 2009.
11. Sun, C. H., C. C. Chiu, C. L. Li, and C. H. Huang, "Time domain image reconstruction for homogenous dielectric objects by dynamic differential evolution," *Electromagnetics*, Vol. 30, No. 4, 309–323, May 2010.
12. Massa, A., M. Pastorino, and A. Randazzo, "Reconstruction of twodimensional buried objects by a hybrid differential evolution method," *Inverse Problems*, Vol. 20, No. 6, 135–150, 2004.
13. Rocca, P., G. Oliveri, and A. Massa, "Differential evolution as applied to electromagnetics," *IEEE Antennas and Propagation Magazine*, Vol. 53, No. 1, 38–49, 2011.
14. Chiu, C. C., C. H. Sun, and W. L. Chang, "Comparison of particle swarm optimization and asynchronous particle swarm optimization for inverse scattering of a two- dimensional perfectly conducting cylinder," *International Journal of Applied Electromagnetics and Mechanics*, Vol. 35, No. 4, 249–261, Apr. 2011.
15. Caorsi, S., M. Donelli, A. Lommi, and A. Massa, "Location and imaging of two-dimensional scatterers by using a Particle Swarm algorithm," *Journal of Electromagnetic Waves and Applications*, Vol. 18, No. 4, 481–494, 2004.
16. Chen, C. H., C. C. Chiu, C. H. Sun, and W. L. Chang, "Two-dimensional finite-difference time domain inverse scattering scheme for a perfectly conducting cylinder," *Journal of Applied Remote Sensing*, Vol. 5, 053522, May 2011.
17. Donelli, M. and A. Massa, "A computational approach based on a particle swarm optimizer for microwave imaging of two-dimensional dielectric scatterers," *IEEE Transactions on Microwave Theory and Techniques*, Vol. 53, No. 5, 1761–1776, Feb. 2006.
18. Donelli, M., G. Franceschini, A. Martini, and A. Massa, "An integrated multi-scaling strategy based on a particle swarm algorithm for inverse scattering problems," *IEEE Transactions on Geoscience and Remote Sensing*, Vol. 44, No. 2, 298–312, 2006.
19. Lee, K. C., "A neural-network-based model for 2D microwave imaging of cylinders," *International Journal of RF and Microwave Computer-aided Engineering*, Vol. 4, No. 5, 398–403, Sep. 2004.
20. Rekanos, I. T., "Shape reconstruction of a perfectly conducting scatterer using differential evolution and particle swarm optimization," *IEEE Transactions on Geoscience and Remote Sensing*, Vol. 46, No. 7, 1967–1974, Jul. 2008.
21. Storn, R. and K. Price, "Differential evolution a simple and efficient adaptive scheme for global optimization over continuous spaces," Technical Report TR-95-012, International Computer Science Institute, Berkeley, 1995.